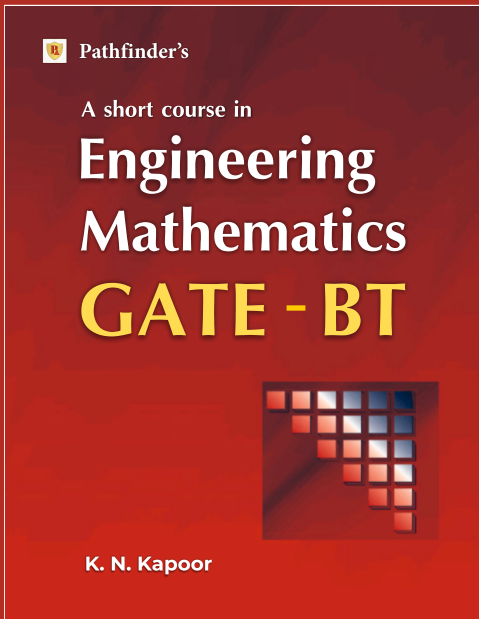
Pathfinder's

A short course in

Engineering Mathematics

GATE - BT

K. N. Kapoor



About the author

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Chapter 01

Linear Algebra

Linear algebra is a branch of Mathematics that deals with linear sets of equations, their transformation properties and theory of matrices. The introduction and development of the notion of a matrix and the subject of linear algebra followed the development of determinants, which arose from the study of coefficients of systems of linear equations. The initial contribution in the theory of matrices was done by the Mathematicians Arthur Caley (1821-1895), James Joseph Sylvester (1814-1847) and S.B. Frobenius (1849-1917).

The matrix theory has been found extensively useful in many branches of applied mathematics such as algebraic and differential equations, Mechanics, theory of electric circuits, nuclear physics, aerodynamics and astronomy. The matrices are also useful for solving coding decoding problems and searching approximate solutions of numerical problems by using computers.

Today *Matrix theory* is one of the most important and powerful tool, not only in Mathematics but also in other disciplines such as Natural and Biological sciences.

1.1 Matrices

A rectangular arrangement of numbers in rows and columns is called matrix. If rectangular array contains *m* rows and *n* column, then we say size of matrix is $m \times n$ (Read it as *m* by *n* not *m* cross *n*).

Matrices are denoted by capital alphabets (A, B, C,….) and their corresponding elements by small alphabets. Let a_{ij} denote, element of matrix A, that belongs to i^{th} row and j^{th} column.

$$A_{mxn} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ : & & & : \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad \text{or,} \quad A = (a_{ij})_{m\times n}; \quad \begin{matrix} i = 1, 2, \dots, m \\ j = 1, 2, \dots, n \end{matrix}$$

1.1.1 Types of matrices

1. **Row matrix :** If matrix contains only one row we call it row matrix.

   The matrix $[-1, 2, 3, 4]_{1 \times 4}$ is row matrix.

2. **Column matrix :** A matrix with only one column is called column matrix.

   The matrix $\begin{pmatrix} 1 \\ 6 \\ 9 \\ -10 \\ 11 \end{pmatrix}_{5 \times 1}$ is a column matrix.

Chapter 02

Calculus

Anything on which mathematical operations such as addition, subtraction, multiplication, division can be performed is called a *quantity*. A quantity which does not change its value (i.e. it always retains the same value) is called an *absolute constant*. Thus 3.41, π, e etc. are absolute constants. A quantity which can take different values for different problems but which retains the same value in a given problem is called an *arbitrary constant*. Thus equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has three arbitrary constants, g, f and c. For different circles, the values of g, f and c are different but they are fixed for a particular circle.

A variable quantity can assume different values in a particular problem. In the above equation of a circle x and y are variables. They are coordinates of a point P(x, y) on the circle and thus change values from point to point. The table given below shows few expression, variable and arbitrary constants.

Expression	*Variable*	*Arbitrary constant*
$ax = b$	x	a, b
$xy = a$	x, y	a
$y^2 = 4ax$	x, y	a
$ax^2 + by^2 = c$	x, y	a, b, c
$ax^2 + bx + c = 0$	x	a, b, c

2.1 Function

Two or more quantities may be related. For example, volume V of a spherical balloon depends on its radius r i.e. $V = \frac{4}{3}\pi r^3$. Such a relationship between two variables is called a function.

In general, if a variable y depends upon another variable x, then y is said to be a function of x and it is written as $y = f(x)$. Here y is called *dependent variable* and x *independent variable*. We shall take real values of x and y. The set of all real values of x for which y has definite, real and finite values is called *domain* of the function $f(x)$. The set of all values (real, definite and finite) for different real and finite values of x is called the *range* of the function. Consider $y = \sqrt{(8 - x)}$. Here y will be real when $(8 - x) \geq 0$ or $x \leq 8$ or which defines the domain of the function y. Further, y can take any real positive value i.e. $y \geq 0$ which is the range of the function.

Chapter 03

Differential equation

3.1 Introduction

An equation involving differentials or derivatives such as $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ etc. is called a *differential equation*.

For example,

1. $(3+\sin x)\frac{dy}{dx}+8\cos y = 5x+8$

2. $2\left(\frac{d^3y}{dx^3}\right)^4 + 5\left(\frac{d^2y}{dx^2}\right)^{11} + 3xy = \tan^2 x$

3. $(x^2+1)\frac{d^5y}{dx^5} + 2x\frac{d^3y}{dx^3} + 13y = 3e^{2x} + 9x + 1$

4. $y = \frac{dy}{dx} + \sqrt{3+\left(\frac{dy}{dx}\right)^2}$

5. $x\frac{dy}{dx} + \frac{2}{dy/dx} = 5y^2 + 4$

6. $\frac{d^2y}{dx^2} + \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 0$

The differential equations arise from many scientific problems such as change of distance of a moving point or body with time, the variation of current in an electric circuit, the oscillations of electrical and mechanical systems and bending of beams etc. A phenomenon obeying a certain law helps in setting up relations between variables, their derivatives and other constants in the form of differential equations. Whenever one variable is changing with respect to another variable, the relationship involving the variables and their rates is likely to be in the form of a differential equation.

3.2 Order and degree of differential equation

The order of a differential equation is the order of the highest order derivative or differential occurring in the equation. The degree of a differential equation is the degree of the highest order derivative in the equation. An equation containing powers of dependent variable or its derivatives or both is called a non-linear differential equation. In the above example, equation (1) and (3) are linear differential equations while all others are non-linear differential equations. Further, equation (1) is of first order and first degree, (2) is of third order and fourth degree, 3 is of fifth order and first degree. Rewrite equation (5) as

Chapter 04

Statistics and Probability

4.1 Statistics

Statistics is as old as the human society itself. Earlier, it was regarded as the science of state-management when government used to collect information about the population of the state such as agriculture, economic conditions, education, military preparation, crimes and various other activities of the state. These information when properly collected, classified and presented to help in the administration and future planning of the state. Statistics deals with the collection, classification, tabulation, analysis of the tabulated data and its interpretation so as to provide a basis for making correct decision.

Population and Sample

A population or universe may be defined as the totality of all actual or conceivable objects under consideration. More accurately, population consists of numerical values connected with these objects. For example, heights of male students of class 12 in Haryana or weight of a student of Delhi University. A population may be finite or infinite. The heights or weights of students, in the above example, constitute finite population. A population containing a finite number of objects or individuals is called a finite population. If the number of objects or individuals are infinite then it is called an infinite population.

A part of the population selected according to some rule is called a sample. The process of selecting sample is called sampling. The number of objects etc. contained in a sample is called sample size. Sometimes, it is not possible or desirable to take into consideration every member of the population. It may be too costly in terms of time and money. Thus we take a sample from a population and examine it to get maximum information's about the parent population. A grain merchant examines only a handful of wheat to know about the quality of it. To find average height of Indians, it is not possible to measure the height of each Indian. We take a sample carefully so that it is truly representative of the population.

When each individual has same chance of selection, then sample is called random sample. We obtain random sample either by lottery system or random number system. After getting a suitable sample from the population we classify and tabulate it. The data is analyzed now statistically. For that we are required to study measures of central tendency (Mean, Median, and Mode etc.), measure of dispersions (mean deviation and standard deviation), correlation etc. Below, we will explain them statistical tools one by one.

Measure of central tendency

When two or more different sets of observations of the same type are compared, it is desirable to find a single number for each set which may be taken as representative for all the observations of that set. This representative number generally lies at the centre of the distribution and it is obtained by taking some average. As such it is said to be an average of a measure of central tendency. A good average should be rigidly defined, based on all the observations taken, calculated easily and true representative of the set of observations. Some commonly used averages are:

Chapter 05

Numerical methods

5.1 Solution of linear simultaneous equations

Simultaneous linear equations occur in various scientific and engineering problems. The standard available methods for the solution of system of linear equations are:

1. Matrix inverse method
2. Linear transformations
3. Cramer's rule

All these methods are discussed in section Linear Algebra.

But these methods become tedious for large systems. However, there exist other numerical methods, which are well-suited for computing machines. The few of available numerical methods are:

1. Gauss elimination methods.
2. Triangular methods.
3. Crout method.

We discuss below, the most commonly used, Gauss elimination methods.

Gauss elimination method

In this method, the unknowns are eliminated successively and the system is reduced to an upper triangular system from which the unknowns are found by back substitution. Here we will explain this method for three variables only. Consider the equation

$$\left.\begin{aligned} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{aligned}\right\} \quad ...(1)$$

Step 1.

First eliminate x from second and third equations.

Assuming $a_1 \neq 0$ we eliminate x from the second equation by subtracting, (a_2/a_1) times the first equation from the second equation. Similarly we eliminate x from the third equation by subtracting (a_3/a_1) times the first equation from the third equations. Thus, we get the new system

$$\left.\begin{aligned} a_1x + b_1y + c_1z = d_1 \\ b_2'y + c_2'z = d_2' \\ b_3'y + c_3'z = d_3' \end{aligned}\right\} \quad ..(2)$$

Here the first equation is called the pivotal equation and a_1 is called the first pivot.

Chapter 06

Laplace Transforms

Laplace transformation of function $f(t)$, defined for all non-negative values of t is denoted by F (s) (or L (f (t)) and is defined as

$$F(s) = L(f(t)) = \int_0^\infty e^{-st} f(t)\, dt \quad : \text{ for all } s \in (a, b),\ a, b \in R$$

provided $f(t)$ is continuous for all values of t and $\lim_{t\to\infty} e^{-st} f(t)$ has some finite value.

Laplace transformation has many applications in different area of mathematics. In differential equation it provides direct solution with boundary values without finding its general solution.

6.1 Linearity property of Laplace transformation

For any arbitrary constants a, b

$$L\left(a\ f(t) + b\ g(t)\right) = a\ L\left(f(t)\right) + b\ L\left(g(t)\right)$$

Proof:

$$L\left(a\ f(t) + b\ g(t)\right) = \int_0^\infty e^{-st}\left(a\ f(t) + b\ g(t)\right) dt$$

$$= a \int_0^\infty e^{-st} f(t)\, dt + b \int_0^\infty e^{-st} g(t)\, dt$$

$$= a\ L\left(f(t)\right) + b\ L\left(g(t)\right)$$

Basic Laplace transforms

1. $L(1) = \dfrac{1}{s}$

2. $L(t^n) = \dfrac{n!}{s^{n+1}}$: $n = 0, 1, 2...$

3. $L(e^{at}) = \dfrac{1}{s-a}$ and $L(e^{-at}) = \dfrac{1}{s+a}$

4. $L(\sin at) = \dfrac{a}{s^2+a^2}$ and $L(\cos at) = \dfrac{s}{s^2+a^2}$

5. $L(\sinh at) = \dfrac{a}{s^2-a^2}$

6. $L(\cosh at) = \dfrac{s}{s^2-a^2}$

7. If $L(f(t)) = F(s)$ then $L\left(e^{at} f(t)\right) = F(s-a)$

Proof:

1. $L(1) = \displaystyle\int_0^\infty e^{-st}.1\, dt = \left(\frac{e^{-st}}{-s}\right)_0^\infty = \frac{1}{s}$: $s > 0$.

GATE Biotechnology

Previous Year's Solved Questions

(Linear algebra)

1. Value of the determinant mentioned below is $\begin{vmatrix} 1 & 0 & -1 & 0 \\ 4 & 7 & 0 & 2 \\ 1 & 1 & -1 & 1 \\ 2 & 0 & 2 & 1 \end{vmatrix}$

a. 24 b. −30

c. −24 d. −10 **GATE 2011**

2. What is the rank of the following matrix?

$$\begin{pmatrix} 5 & 3 & -1 \\ 6 & 2 & -4 \\ 14 & 10 & 0 \end{pmatrix}$$

a. 0 b. 1

c. 2 d. 3 **GATE 2012**

3. The solution to the following set of equation is

$2x + 3y = 4$

$4x + 6y = 0$

a. x = 0, y = 0 b. x = 2, y = 0

c. 4x = −6y d. no solution **GATE 2014**

4. One of the eigenvalues of $P = \begin{pmatrix} 10 & -4 \\ 18 & -12 \end{pmatrix}$ is

a. 2 b. 4

c. 6 d. 8 **GATE 2013**

5. If $P = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, Q = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$ and $R = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$, which one of the following statements is *true*?

a. PQ = PR b. QR = RP

c. QP = RP d. PQ = QR **GATE 2013**

6. The solution for the following set of equations is,

$5x + 4y + 10z = 13$

$x + 3y + z = 7$

$4x - 2y + z = 0$

a. x = 2, y = 1, z = 1 b. x = 1, y = 2, z = 0

c. x = 1, y = 0, z = 2 d. x = 0, y = 1, z = 2 **GATE 2014**